AP Calculus BC Prerequisite Notes for Summer Review Packet for students who completed AP Calculus AB

These notes are designed to help you understand how to accurately complete the Summer Review Packet. Go to my website **www.mrdemsey.com** ,then click on "AP Calculus BC" and then click on "Prerequisite Chapter." Under the heading "Notes Videos," when you click on the link for each Section, you will be directed to my videos so you can see and hear my explanation of how to solve the example problems in order to complete the notes in this packet. The first exam of the year will be mostly based on these notes and the problems in the Summer Review Packet. If you have questions as you work on the packet, you can email me anytime at **toshdemsey@cusd.com. .** I will also have four optional sessions during the summer where I can help you out. You can also attend one or more of the optional tutoring sessions during the summer that will take place in my classroom, D06, on Friday August 11th from 2 p.m. to 4 p.m., on Saturday August 12th from 2 p.m. to 4 p.m., on Sunday August 13th from 2 p.m. to 4 p.m., and on Saturday August 19th from 2 p.m. to 4 p.m. To receive text messages from me via Remind, please send a text message to 81010 and type in the message @mrdcalcbc If you have not previously used Remind for another teacher, you will receive a text message back that requests that you type in your name. You will earn 20 points of extra credit for signing up for Remind.

Section A – Solving Absolute Value Inequalities

Recall: **absolute value is the distance away from zero on the number line.**

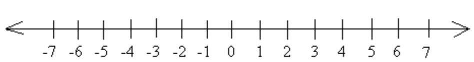
Steps For Solving Linear Absolute Value Inequalities

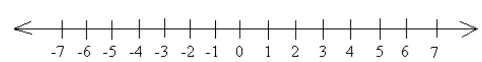
1) Isolate the absolute value by getting the absolute value by itself on one side of the inequality.
2) Draw a sketch on the number line that represents the correct distance away from zero.
3) If the inequality sign is $<$ or $\leq$, rewrite as a compound inequality and solve by isolating the variable.
4) If the inequality sign is $>$ or $\geq$, rewrite as two inequalities and solve each by isolating the variable.
5) Graph the solution on the number line.

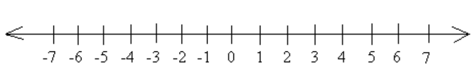
Examples: Solve the following inequalities and graph the solution.

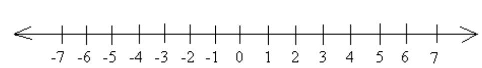
$|x-2| \leq 1$

$\dfrac{|x|}{3} < 2$



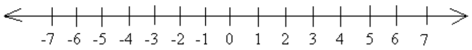


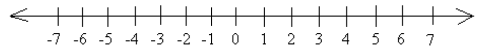


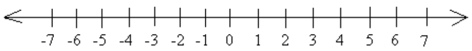


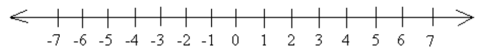
$|x+1| \geq 3$

$-2|2x-1| < -4$









<u>Steps For Solving Non-Linear Absolute Value Inequalities</u>

1) Isolate the absolute value by getting the absolute value by itself on one side of the inequality.
2) If the absolute value has no effect on the expression inside it, then simplify and get one side of the inequality equal to zero.
3) Factor completely and set each factor equal to zero.
4) Solve each of the equations from step 3, and put the solutions on the number line.
5) Test a number on each interval between the solutions found in step 4 in the factored form to determine whether the interval is positive or negative.
6) Graph the solution on the number line based on whether the inequality was looking for positive or negative solutions.

Examples: Solve the following inequalities and graph the solution.

$\frac{|x^2|}{4} < 16$

$|(x-1)^2| \leq 1$

$-5|(x+3)^2| > -5$

$|x^4| + 3 \leq 19$

Section B – Parametric Equations

If f and g are continuous functions on an interval, then the equations

$x = f(t)$ and $y = g(t)$ are called parametric equations and t is the parameter.

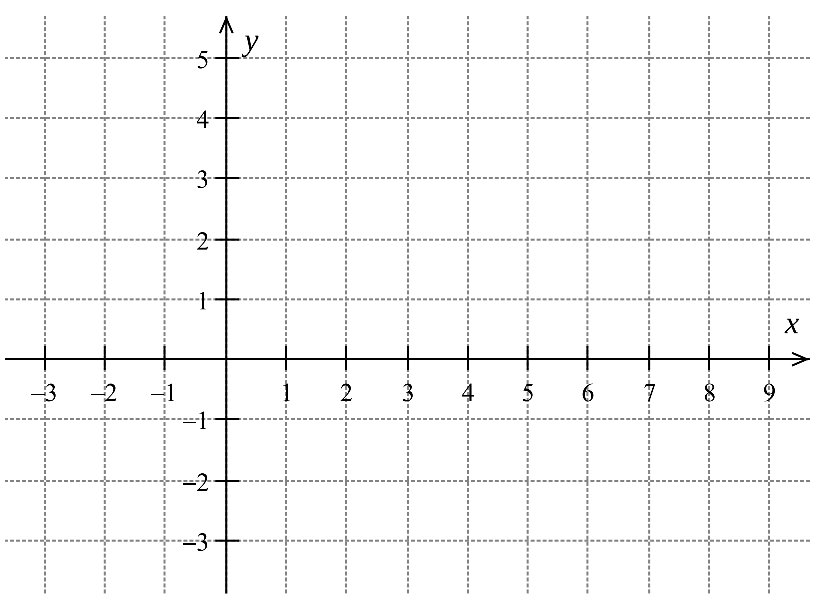
For $t \geq 0$, a particle is moving along a curve so that the position of the particle at any time t is given by

$x(t) = t^3 - 2t^2 + t - 3$ $\qquad$ $y(t) = t^2 - 2t + 1$

a) Complete the table below:

t	0	1	2	3
x				
y				

b) Sketch the path of the particle in the xy-plane below. Indicate the direction of motion along the path.



c) At $t = \frac{1}{2}$, is the particle moving to the left or to the right? Justify your answer.

d) At $t = \frac{1}{2}$, is the particle moving up or down? Justify your answer.

e) At what point(s) (x, y), if any, is the particle at rest?

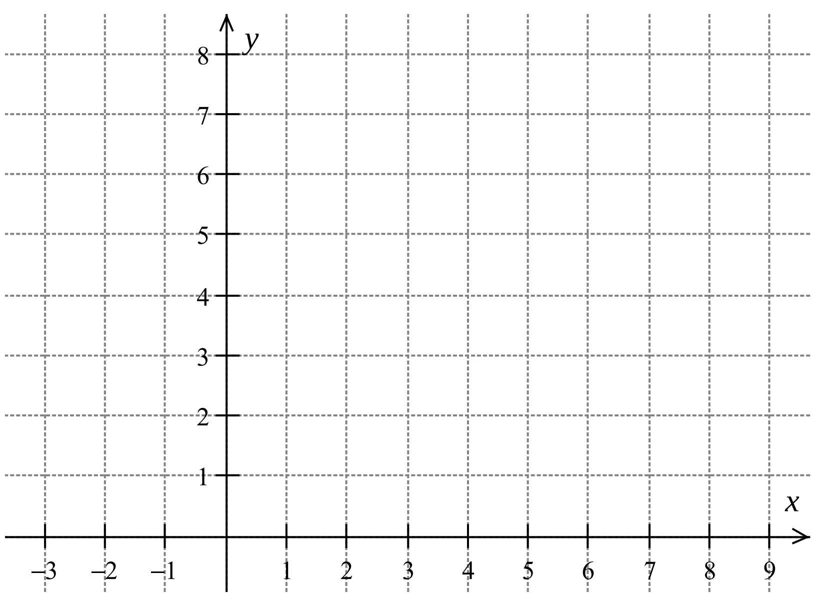
For $t \geq 0$, a particle is moving along a curve so that the position of the particle at any time t is given by

$x = t^2 - 4t + 3$ $\qquad\qquad$ $y = \left(\sqrt{t} - 1\right)^3$

a) Complete the table below:

t	0	1	2	3
x				
y				

b) Sketch the path of the particle in the *xy*-plane below. Indicate the direction of motion along the path.



c) At $t = 3$, is the particle moving to the left or to the right? Justify your answer.

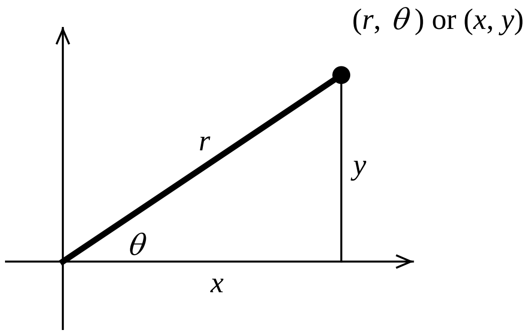
d) At $t = 3$, is the particle moving up or down? Justify your answer.

e) At what point(s) (x, y), if any, is the particle at rest?

Section C – Polar Equations

Each point P in the plane can be assigned polar coordinates $(r,\ \theta)$ where r is the distance from the origin to point P and Θ is the directed angle to point P measured in radians.

To convert back and forth between rectangular coordinates and polar coordinates use the following:



$x = r\cos\theta$ $\qquad$ $x^2 + y^2 = r^2$

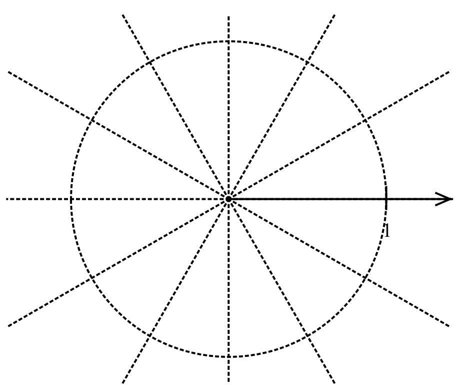
$y = r\sin\theta$ $\qquad$ $\tan\theta = \dfrac{y}{x}$

Example: Given the polar curve $r = 1 - 2\sin\theta$ for $0 \le \theta \le \pi$

a) Write expressions for x and y in terms of θ.

b) Complete tables below and use them to sketch the graph of the polar curve $r = 1 - 2\sin\theta$ for $0 \le \theta \le \pi$

θ	r
0	
$\frac{\pi}{6}$	
$\frac{\pi}{4}$	
$\frac{\pi}{3}$	
$\frac{\pi}{2}$	
$\frac{2\pi}{3}$	
$\frac{3\pi}{4}$	
$\frac{5\pi}{6}$	
π	

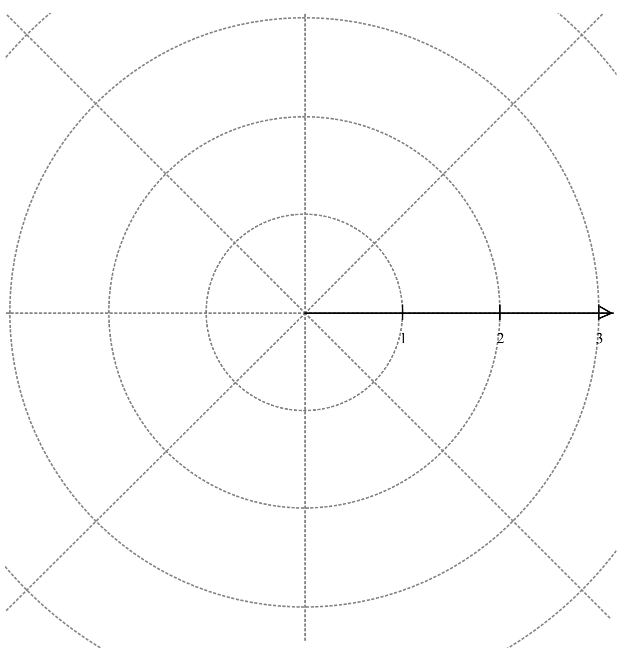


Example: Given the polar curve $r = 2\cos^2\theta$ for $0 \le \theta \le 2\pi$

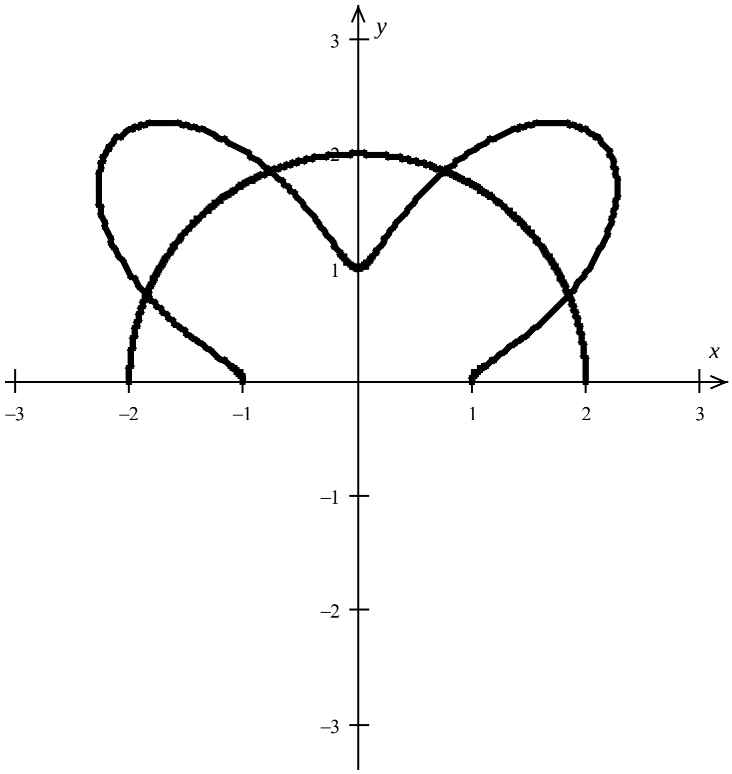
a) Write expressions for x and y in terms of θ.

b) Complete tables below and use them to sketch the graph of the polar curve $r = 2\cos^2\theta$ for $0 \le \theta \le 2\pi$

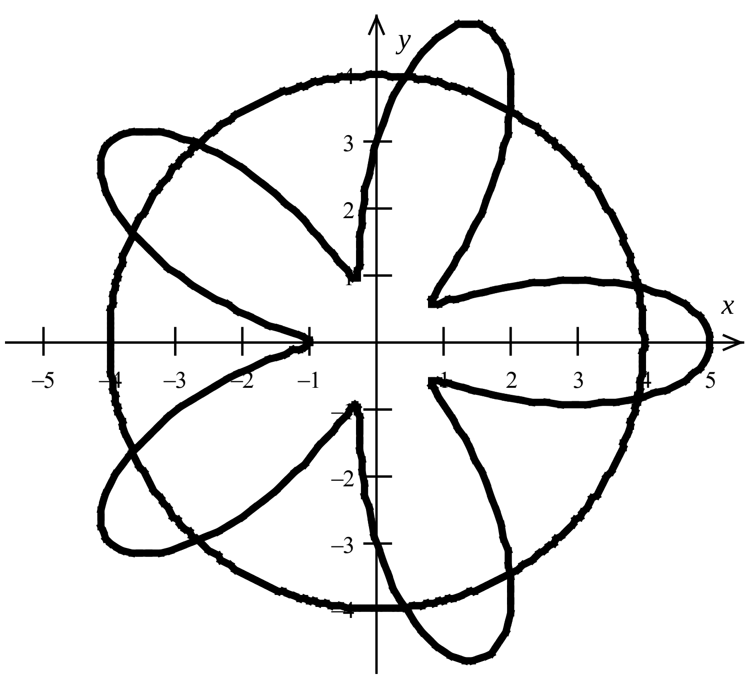
θ	r
0	
$\frac{\pi}{4}$	
$\frac{\pi}{2}$	
$\frac{3\pi}{4}$	
π	
$\frac{5\pi}{4}$	
$\frac{3\pi}{2}$	
$\frac{7\pi}{4}$	
2π	



Example: Given the graphs of the polar curves $r = 2\sin^2(2\theta)+1$ and $r = 2$ for $0 \le \theta \le \pi$, find all the angles, θ, where the graphs intersect and label them on the graph.



Example: Given the graphs of the polar curves $r = 2\cos(5\theta)+3$ and $r = 4$ for $0 \le \theta \le 2\pi$, find all the angles, θ, where the graphs intersect and label them on the graph.



Section D – Infinite Geometric Series

A series is a sum of a sequence.

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + a_4 + ...$$

Example:

$$\sum_{n=1}^{\infty} 3(2)^{n-1}$$

Partial sums are finite series

$$S_1 = \sum_{n=1}^{1} a_n = a_1$$

$S_1 =$

$$S_2 = \sum_{n=1}^{2} a_n = a_1 + a_2$$

$S_2 =$

$$S_3 = \sum_{n=1}^{3} a_n = a_1 + a_2 + a_3$$

$S_3 =$

How to tell if an Infinite Geometric Series converges or diverges

For an infinite geometric series, $\sum_{n=1}^{\infty} a(r)^{n-1}$, the series converges if $-1 < r < 1$ and has a sum $S = \frac{a_1}{1-r}$.

Examples: Find the first three partial sums of the series. State whether the series converges or diverges. If the series converges, find its sum.

$$\sum_{n=1}^{\infty} 4\left(-\frac{1}{2}\right)^{n-1}$$

$$\sum_{n=1}^{\infty} \frac{3^{2n-1}}{2 \bullet 3^n}$$

Example: Find the common ratio, *r*, and, determine the *x*-values for which the series converges, and then find its sum.

$$5(x-1) + 10(x-1)^2 + 20(x-1)^3 + ...$$

$$2 - 8(x-3) + 32(x-3)^2 + ...$$

AP Calculus BC Summer Review Packet
(80 homework points – Due Monday 8/21/17, 20 points extra credit if completed and turned in by 8/19/17)

This packet is designed to help you review and build upon some of the important mathematical concepts and skills that you have learned in your previous mathematics classes that you will be using in AP Calculus BC. Before you work on each section, go to my website www.mrdemsey.com and watch the corresponding videos and fill out the prerequisite notes for that section. The first exam of the year will be on Friday 8/25/17 and will be based on the concepts and methods you will be practicing in the problems of this packet.

SHOW ALL WORK ON PACKET OR SEPARATE SHEETS OF PAPER

Section A – Solving Absolute Value Inequalities

Solve the following inequalities and graph the solution.

1. $|x+1|<2$
2. $-2|x|\geq -8$
3. $3|x-4|>9$
4. $\frac{|x+2|}{3}<1$
5. $|3x-1|\geq 4$
6. $|2x|<3$
7. $2|x^2|>18$
8. $|(x+2)^2|<1$
9. $|(x-3)^2|\leq 9$
10. $\frac{|x^2|}{3}\geq 12$
11. $|x^4|+1<82$
12. $3-2|x^2|\geq 1$

Section B – Parametric Equations

1. For $t\geq 0$, a particle is moving along a curve so that the position of the particle at any time t is given by

$x(t)=t^3-9t^2+24t+1$ $\quad\quad$ $y(t)=t^2-8t-1$

a) Complete the table below:

t	0	1	2	3
x				
y				

b) Sketch the path of the particle in the xy-plane. Indicate the direction of motion along the path.
c) At t = 1, is the particle moving to the left or to the right? Justify your answer.
d) At t = 1, is the particle moving up or down? Justify your answer.
e) At what point(s) (x, y), if any, is the particle at rest?

2. For $t \geq 0$, a particle is moving along a curve so that the position of the particle at any time t is given by

$$x(t) = \left(\sqrt{5-t}\right)^3 \qquad y(t) = 2 + 6t^2 - t^3$$

a) Complete the table below:

t	0	1	2	3	4
x					
y					

b) Sketch the path of the particle in the *xy*-plane. Indicate the direction of motion along the path.
c) At $t = 3$, is the particle moving to the left or to the right? Justify your answer.
d) At $t = 3$, is the particle moving up or down? Justify your answer.
e) At what point(s) (x, y), if any, is the particle at rest?

3. For $t \geq 0$, a particle is moving along a curve so that the position of the particle at any time t is given by

$$x = t^3 - 9t^2 + 2 \qquad y = t^3 - 15t^2 + 72t - 4$$

a) Complete the table below:

t	0	1	2	3
x				
y				

b) Sketch the path of the particle in the *xy*-plane. Indicate the direction of motion along the path.
c) At $t = 3$, is the particle moving to the left or to the right? Justify your answer.
d) At $t = 3$, is the particle moving up or down? Justify your answer.
e) At what point(s) (x, y), if any, is the particle at rest?

4. For $t \geq 0$, a particle is moving along a curve so that the position of the particle at any time t is given by

$$x = \cos t \qquad y = \sin t$$

a) Complete the table below:

t	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x					
y					

b) Sketch the path of the particle in the *xy*-plane. Indicate the direction of motion along the path.
c) At $t = \frac{3\pi}{2}$, is the particle moving to the left or to the right? Justify your answer.
d) At $t = \pi$, is the particle moving up or down? Justify your answer.
e) At what point(s) (x, y), if any, is the particle at rest?

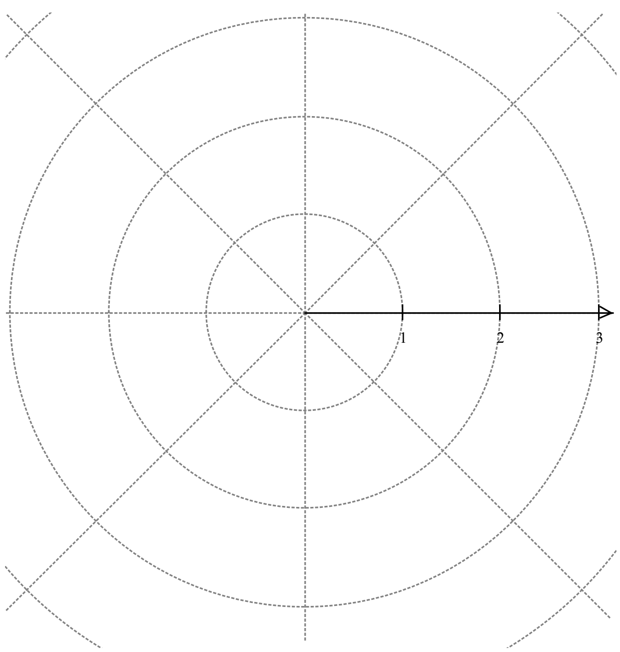
Section C – Polar Equations

1. Given the polar curve $r = 1 + 2\cos\theta$ for $0 \le \theta \le \pi$

a) Write expressions for x and y in terms of θ.

b) Complete table below and use them to sketch the graph of the polar curve $r = 1 + 2\cos\theta$ for $0 \le \theta \le \pi$

θ	r
0	
$\frac{\pi}{6}$	
$\frac{\pi}{4}$	
$\frac{\pi}{3}$	
$\frac{\pi}{2}$	
$\frac{2\pi}{3}$	
$\frac{3\pi}{4}$	
$\frac{5\pi}{6}$	
π	

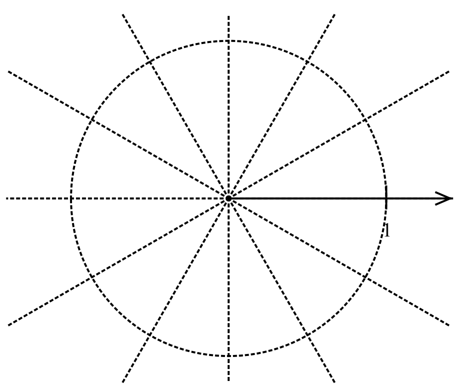


2. Given the polar curve $r = \sin^2\theta$ for $0 \le \theta \le 2\pi$

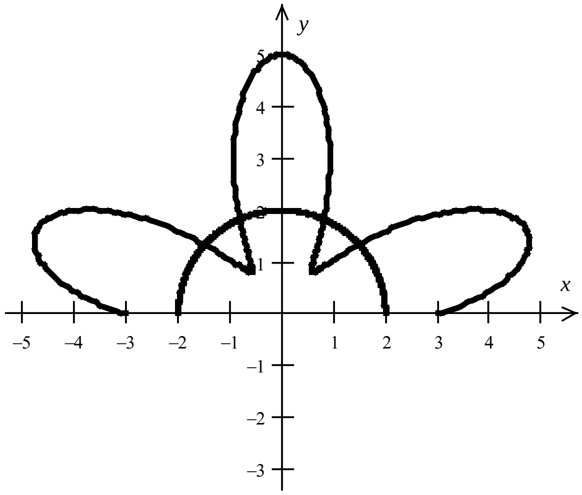
a) Write expressions for x and y in terms of θ.

b) Complete table below and use them to sketch the graph of the polar curve $r = \sin^2\theta$ for $0 \le \theta \le 2\pi$

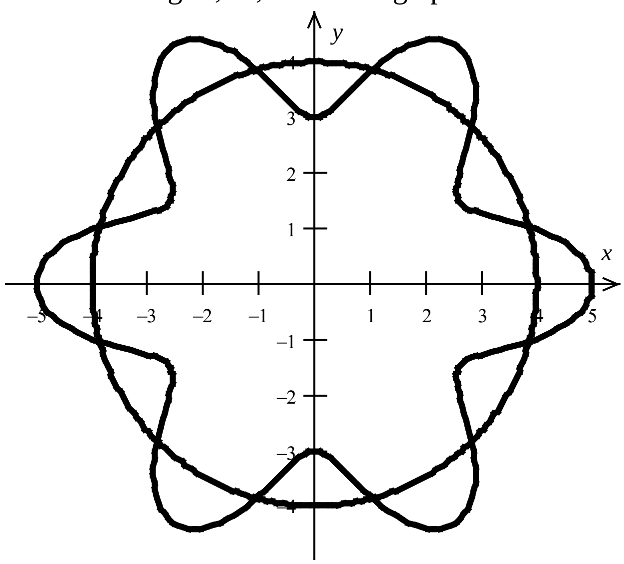
θ	r
0	
$\frac{\pi}{4}$	
$\frac{\pi}{2}$	
$\frac{3\pi}{4}$	
π	
$\frac{5\pi}{4}$	
$\frac{3\pi}{2}$	
$\frac{7\pi}{4}$	
2π	



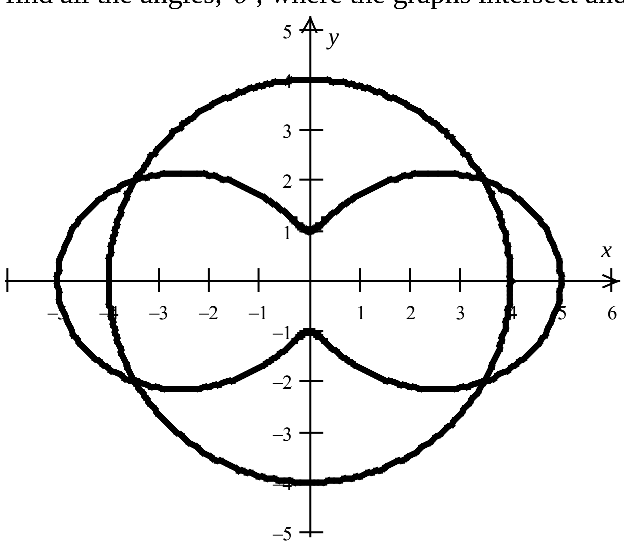
3. Given the graphs of the polar curves $r = 2\sin(5\theta) + 3$ and $r = 2$ for $0 \le \theta \le \pi$, find all the angles, θ, where the graphs intersect and label them on the graph.



4. Given the graphs of the polar curves $r = 2\cos^2(3\theta) + 3$ and $r = 4$ for $0 \le \theta \le 2\pi$, find all the angles, θ, where the graphs intersect and label them on the graph.



5. Given the graphs of the polar curves $r = 4\cos^2(\theta) + 1$ and $r = 4$ for $0 \le \theta \le 2\pi$, find all the angles, θ, where the graphs intersect and label them on the graph.



Section D – Infinite Geometric Series

Find the first three partial sums of the series. State whether the series converges or diverges. If the series converges, find its sum.

1. $\sum_{n=1}^{\infty}\left(-\frac{3}{4}\right)^{n-1}$

2. $\sum_{n=1}^{\infty}\frac{2^{n+1}}{4}$

3. $\sum_{n=1}^{\infty}\frac{2}{5^{n-1}}$

4. $\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^{n+1}$

5. $\sum_{n=1}^{\infty}\left(-\frac{3}{2}\right)^{n-1}$

6. $\sum_{n=1}^{\infty}\left(\frac{e}{\pi}\right)^{n-1}$

7. $\sum_{n=1}^{\infty}\frac{2^{n+1}}{3^n}$

8. $\sum_{n=1}^{\infty}\frac{3^n}{4^{n-1}}$

Find the common ratio, r, and, determine the x-values for which the series converges, and then find its sum.

9. $3(x+1)+12(x+1)^2+48(x+1)^3+\ldots$

10. $(x-2)+7(x-2)^2+49(x-2)^3+\ldots$

11. $4+8(x+3)+16(x+3)^2+\ldots$

12. $4-12(2x-1)+36(2x-1)^2+\ldots$