**AP Calculus AB Prerequisite Notes for Summer Review Packet**

These notes are designed to help you understand how to accurately complete the Summer Review Packet. Go to my website **www.mrdemsey.com** ,then click on "AP Calculus AB," and then click on "Prerequisite Chapter." Under the heading "Notes Videos," when you click on the link for each Section, you will be directed to my videos so you can see and hear my explanation of how to solve the example problems in order to complete the notes in this packet. The first exam of the year will be mostly based on these notes and the problems in the Summer Review Packet. If you have questions as you work on the packet, you can email me anytime at **toshdemsey@cusd.com.** I will also have four optional tutorial sessions during the summer where I can help you out. You can attend one or more of the optional tutoring sessions during the summer that will take place in my classroom, D06, on Friday August 11th from 2 p.m. to 4 p.m., on Saturday August 12th from 2 p.m. to 4 p.m., on Sunday August 13th from 2 p.m. to 4 p.m., and on Saturday August 19th from 2 p.m. to 4 p.m. To receive text messages from me via Remind, please send a text message to 81010 and type in the message @mrdcalcab If you have not previously used Remind for another teacher, you will receive a text message back that requests that you type in your name. You will earn 20 points of extra credit for signing up for Remind.

## Section A - Linear Equations

A linear equation represents a relationship between two variables with a constant rate of change (slope). Since a linear equation has a constant rate of change (slope), its graph is a straight line.

Slope

The slope, *m*, of a straight line that passes through points $(x_1, y_1)$ and $(x_2, y_2)$

$$m = \frac{change \quad in \quad y-coordinate}{change \quad in \quad x-coordinate} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \text{ or } \frac{y_1 - y_2}{x_1 - x_2}$$

Two of the most frequently used forms of linear equations are:

Point-slope form

$y - y_1 = m(x - x_1)$ where $m$ is the slope and $(x_1, y_1)$ is any point on the line.

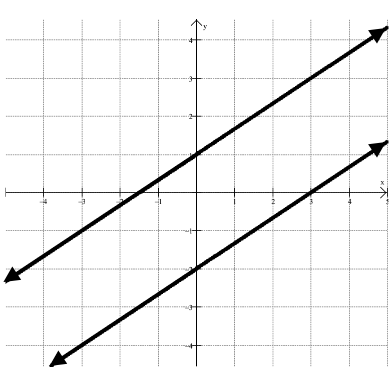
Slope-intercept form

$y = mx + b$ where $m$ is the slope and $b$ is the y-intercept.

Parallel Lines

Parallel lines have the same slope and do not intersect.

$y = \frac{2}{3}x + 1$

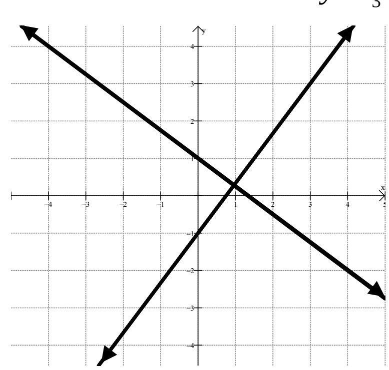


$y = \frac{2}{3}x - 2$

Perpendicular Lines

Perpendicular lines have slopes that are opposite reciprocals and intersect at a right (90°) angle.

$y = \frac{4}{3}x - 1$

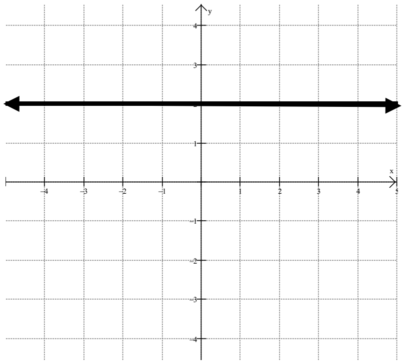


$y = -\frac{3}{4}x + 1$

## Horizontal Lines

Horizontal lines have a slope of zero since there is no change in the y-coordinate which yields a slope of $\frac{0}{n} = 0$

Since a horizontal line has a constant y-coordinate, its equation is in the form $y = c$, where *c* is a constant.



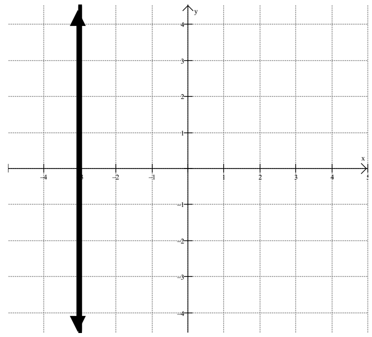
$y = 2$

## Vertical Lines

Vertical lines have an undefined slope since there is no change in the x-coordinate which yields a slope of $\frac{n}{0}$ undefined

Since a vertical line has a constant x-coordinate, its equation is in the form $x = c$, where *c* is a constant.

$x = -3$



Examples: Based on the information given, write the linear equation in:

a) point-slope form

b) slope-intercept form

The line with a slope of 4 that passes through the point $(7, 2)$

The line that passes through the points (-2, 7) and (3, -5)

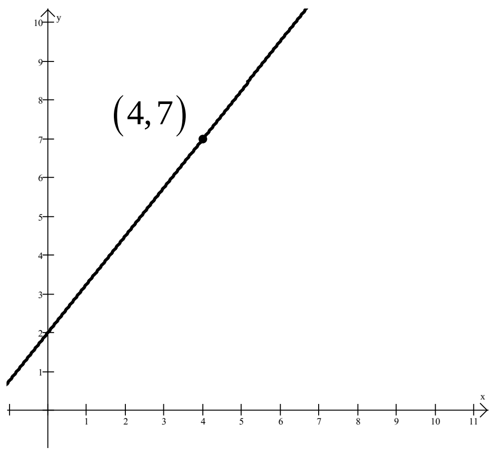
The line with an x-intercept of -2 that is parallel to $3x + 4y = 3$

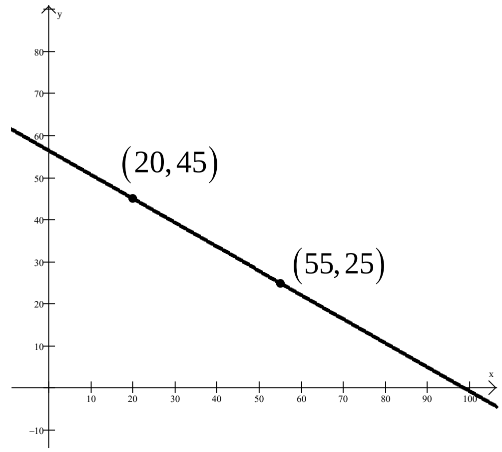
The line perpendicular to $9y - 15x = 8$ containing the point $(9, 4)$

A vertical line that goes through the point $(-3, 4)$ a) What is the slope of the line?

b) Write equation of the line. c)Write the equation of the line perpendicular to the line that passes through the point $(2, -8)$

Write the linear equation in slope-intercept form using the given graph.




The functions *f* and *g* are continuous functions that have the values given by the tables below. Determine whether each function could be a linear equation and explain why or why not.

a)

| $x$ | $f(x)$ |
|---|---|
| 4 | 20 |
| 7 | 18 |
| 13 | 14 |

b)

| $x$ | $g(x)$ |
|---|---|
| 1 | 4 |
| 3 | 10 |
| 9 | 21 |

## Section B Factoring , Simplifying, and Solving Equations and Inequalities Using Factoring

Examples: Factor completely.

$81x - x^5$

$3x^4 - 5x^3 - 12x^2$

Examples: Simplify each expression using factoring.

$$\frac{15-3t}{t^2-25}$$

$$\frac{x(4-3x)+4(x^2+1)}{x^3-4x}$$

Examples: Solve the following equations by factoring.

$6x^2 - 11x = -4$

$(2x-1)^2(x+3)^2 + (x+3)(2x-1)^3 = 0$

Examples: Solve the following inequalities by factoring. Write your answer using inequality and interval notation.

$x^2 + x \ge 2$

$\dfrac{x^2 - 4}{x^2 - x - 20} < 0$

## Section C Exponents, Radicals, and Simplifying

A fractional exponent means you are taking a root. For example $x^{1/2}$ is the same as $\sqrt{x}$.

The rule for switching from a rational exponent to radical form is $x^{\frac{a}{b}} = \left(\sqrt[b]{x}\right)^a$

The rules for rewriting a negative exponent are $x^{-a} = \dfrac{1}{x^a}$ and $\dfrac{1}{x^{-a}} = x^a$

Examples: Write without fractional or negative exponents.

$y = x^{2/3}$

$y = 2x^{-4}$

$f(x) = 8^{-\frac{2}{3}} 32^{\frac{3}{5}} x^{\frac{3}{4}}$

$y = \dfrac{3x^{-\frac{4}{5}}}{(x+2)^{-\frac{1}{2}}}$

Examples: Write without fractional or negative exponents in order to find the following:

$h(x) = 18(x+1)^{-\frac{3}{2}} - x^{\frac{2}{3}}$ $\quad h(8) =$

$v(t) = \dfrac{4(t-1)^{\frac{1}{3}}}{3} + \dfrac{4t^{\frac{3}{2}}}{9}$ $\quad v(9) =$

When factoring, always factor out the lowest exponent for each common term. Remember, the remaining exponent must add with the exponent of the term factored out to equal the original exponent because of the rule $x^a x^b = x^{a+b}$

Examples: Factor and then simplify.

$y = 3x^{-2} + 6x - 33x^{-1}$

$f(x) = 4x(x-3)^{1/2} + x^2(x-3)^{-1/2}$

Examples: Demonstrate that the two sides of the equation are equivalent by simplifying only the **left side** to make it look identical to the right side.

$$-\frac{1}{2}x\left(x^2+1\right)^{-\frac{3}{2}}(2x)+\left(x^2+1\right)^{-\frac{1}{2}}=\frac{1}{\left(\sqrt{x^2+1}\right)^3}$$

## Section D Manipulating Equations and Solving Systems of Equations

Examples: Solve for *y* in terms of the other variable(s).

$x^2 + 3xy - 8x - 4y = 0$

$2\ln\left(y-4\right)-1=3x^2-17$

$3e^y + 5 = 4x$

$\frac{1}{y^2+3} = x^3 + 2x - 5$

$\tan\left(3y+2\right)=x^2+5$

Examples: Solve the system of equations using substitution or elimination.

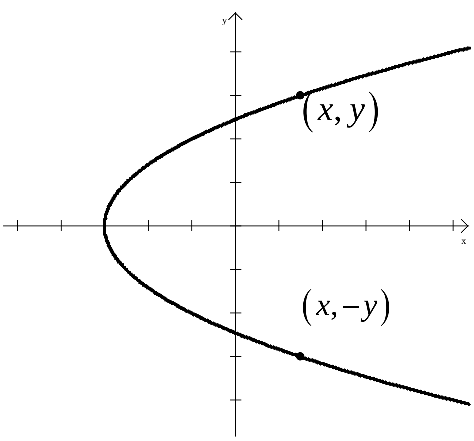
$$3x+4y=1$$
$$-6x+2y=3$$

$$3a-5b=-8$$
$$a=1-2b$$

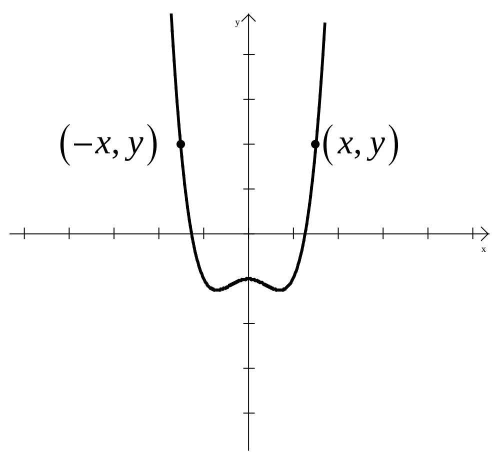
$$y=\sqrt{x-2}$$
$$y=x-2$$

## Section E Symmetry and Intercepts

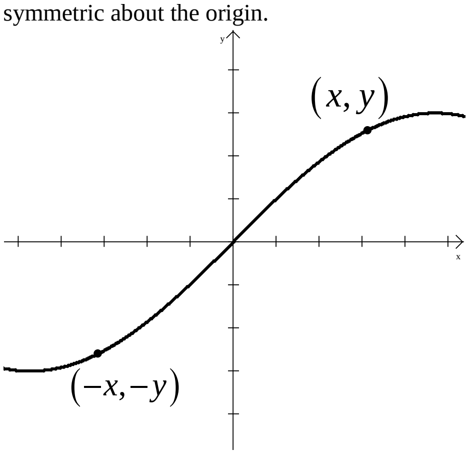
Symmetry about the *x*-axis (with respect to the *x*-axis)
If a graph is symmetric to the x-axis for every point (*x*,*y*) on the graph the point (*x*,-*y*) is also on the graph.
To test for symmetry about the x-axis substitute in –*y* for *y* into the equation. If this yields an equivalent equation, then it is symmetric about the *x*-axis.




Symmetry about the *y*-axis (with respect to the *y*-axis)
If a graph is symmetric to the y-axis for every point (*x*,*y*) on the graph the point (-*x*,*y*) is also on the graph.
To test for symmetry about the *y*-axis substitute in –*x* for *x* into the equation. If this yields an equivalent equation, then it is symmetric about the *y*-axis.




Symmetry about the origin (with respect to the origin)
If a graph is symmetric to the origin for every point (*x*,*y*) on the graph the point (-*x*,-*y*) is also on the graph. To test for symmetry about the origin substitute in –*x* for *x* into the equation and substitute in –*y* for *y* into the equation. If this yields an equivalent equation, then it is symmetric about the origin.



Example: Check for symmetry with respect to each axis and the origin.

$$xy - \sqrt{4 - x^2} = 0$$

<u>x-intercepts</u>

The *x*-intercept(s) are where the graph touches the *x*-axis. Since the *y*-coordinate of any *x*-intercept is 0, you can find the *x*-intercept by setting *y*=0 and then solving for *x*.

<u>y-intercepts</u>

The *y*-intercept(s) are where the graph touches the *y*-axis. Since the *x*-coordinate of any *y*-intercept is 0, you can find the *y*-intercept by setting *x*=0 and then solving for *y*.

Examples: Find the *x* and *y* intercepts for each of the following:

$y = (x+3)^2 - 4$

$x^2 + x^2 y + 3y - 4 = 0$

# <u>Section F - Functions</u>

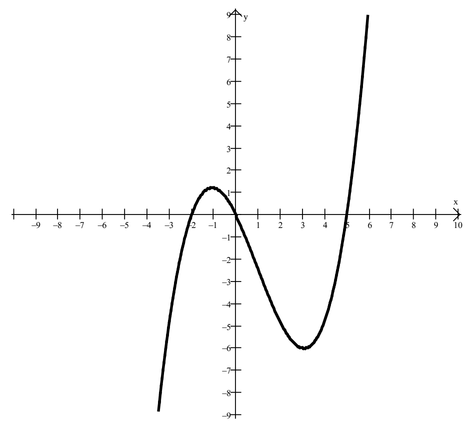
A function is a specific type of relation in which each independent variable (often $x$ or $t$), in the domain corresponds to one and only one dependent variable (often $y$) in the range.

The <u>zeros of a function</u> are the values of the independent variable, if any, that make the function equal to zero. Graphically, they look just like $x$-intercepts, and algebraically they are found using the same technique. The only difference is that you set the function, such as $f(x)$, equal to zero, rather than $y$.

Examples: Find the zeros of the function.

$f(x) = 3x^4 - 48$

$g(x)$



<u>Function Notation and Composite Functions</u>

If a number or expression replaces the independent variable, then that number or expression should be substituted into the function for the independent variable.

The function given by $(f \circ g)(x) = f(g(x))$ is called the composite of $f$ with $g$. The domain of $f \circ g$ is the set of all $x$ in the domain of $g$ such that g(x) is in the domain of $f$.

$(f \circ g)(x) = f(g(x))$ $\quad$ $(g \circ f)(x) = g(f(x))$ $\quad$ $(f \circ f)(x) = f(f(x))$

Example: Given: $f(x) = 3x + 5$ and $g(x) = x^2 + x - 12$, find each of the following:

a) What are the zeros of $g$ ?

b) $f(g(2))$

c) $(g \circ f)(-2)$

d) $g(f(x))$

e) $(f \circ g)(x)$

f) $g(\ln x)$

Examples: Use the table below and $h(0) = -3$, $h(1) = -1$, $h(3) = 0$ to find the following:

| $x$ | -5 | -3 | -1 | 0 | 1 | 3 | 5 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 12 | 9 | 7 | 5 | 3 | 1 | -2 |
| $g(x)$ | 5 | 3 | 0 | -1 | -3 | -5 | -8 |

a) $f(g(-1))$

b) $h^{-1}(0)$

c) $(h \circ f)(3)$

d) $f(g^{-1}(-5))$

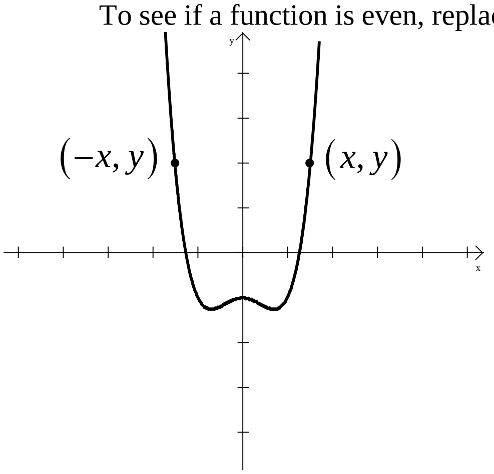
e) $f(g(g(-3)))$

f) $f^{-1}(g(h(0)))$

Even and Odd Functions

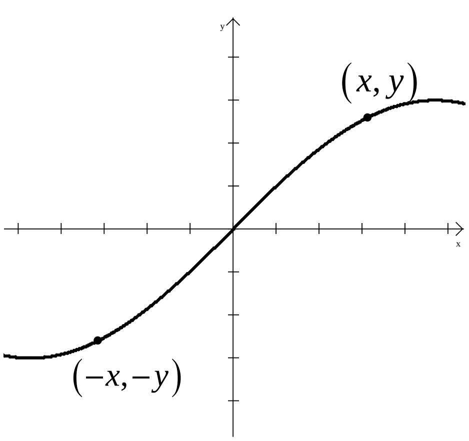
**Even functions are symmetrical about the y-axis.**

To see if a function is even, replace x with -x, and see if expression is the same.




**Odd functions are symmetrical about the origin.**

To see if a function is odd, replace x with –x, and see if the expression is the opposite.



Examples: Show whether the functions are even, odd, or neither.

$f(x) = x^4 + 2x - 2$ $\qquad$ $h(x) = \dfrac{x^4}{x^2 - 4}$ $\qquad$ $j(x) = -2x^3 + 4x$

## Domain and Range

The domain of a function is the set of values of the independent variable (often $x$ or $t$) for which the function is defined. The range of a function is the set of values of the dependent variable (often $y$) that a function can return. In calculus, we will often write domains and ranges in interval notation. If the domain were $-1 < x \le 7$ then in interval notation the domain would be (-1,7]. Notice that the left side has a ( because it does not include -1 but the right side includes 7 so we use a ]. When using interval notation we never use a [ or ] for infinity.

To find the domain, look for restrictions on the independent variable.

Common restrictions:

1) Denominators of fractions cannot be zero.
2) Expressions inside radicals (with an even root) must be greater than or equal to zero.
3) Expressions inside logarithms must be greater than zero.

Examples: Find the domain of the following functions.

$f(x) = \dfrac{\sqrt{x+5}}{2x}$ $\qquad$ $g(x) = \dfrac{x+1}{x^2 - x - 12}$

$h(x) = \ln(3 - x)$ $\qquad$ $j(x) = \dfrac{x+2}{\sqrt{9x^2 - 4}}$

Inverse Functions

$f^{-1}(x)$ is the notation used to represent the inverse function of the function $f(x)$.

**Property 1**

$f(f^{-1}(x)) = x$ $\qquad$ $f^{-1}(f(x)) = x$

**Property 2**

The domain of $f(x)$ is the range of $f^{-1}(x)$.

The range of $f(x)$ is the domain of $f^{-1}(x)$.

**Property 3**

If $f(x)$ contains the point (a,b) then $f^{-1}(x)$ contains the point (b,a).

This means the inverse functions are symmetrical about the line $y = x$

**Existence of an Inverse Function**

A function has an inverse if it is **strictly monotonic** on its entire domain (always increasing or always decreasing)

The **horizontal line test**: To have an inverse, the function must not cross any horizontal line more than once. This means it is **one-to-one** (For every *y*-value there is one and only one corresponding *x*-value.)

**Finding an Inverse Function**

1) Let $f(x) = y$
2) Switch the *x* and *y* variables.
3) Solve for *y*.
4) Let $y = f^{-1}(x)$

Examples: Find the inverse of each function.

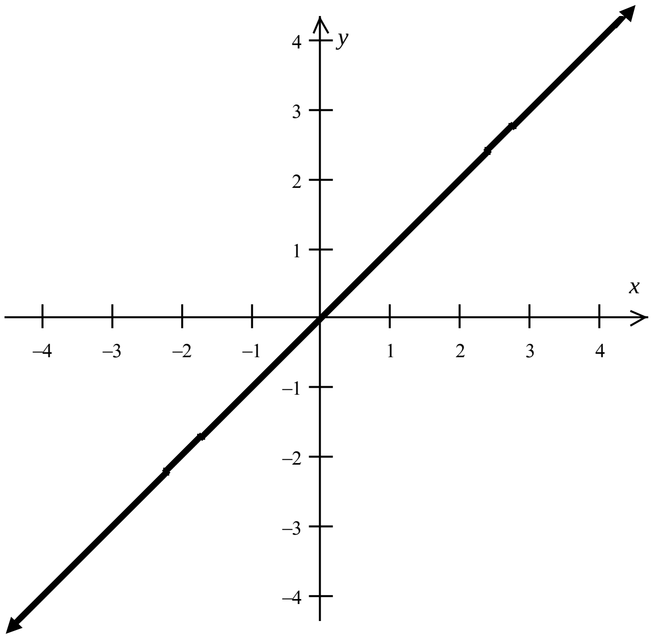
$f(x) = \dfrac{3x+2}{x-5}$ $\qquad$ $g(x) = \sqrt{x^2 - 1}$

$h(x) = \ln(x+5)$ $\qquad$ $j(x) = 2e^{x+4}$

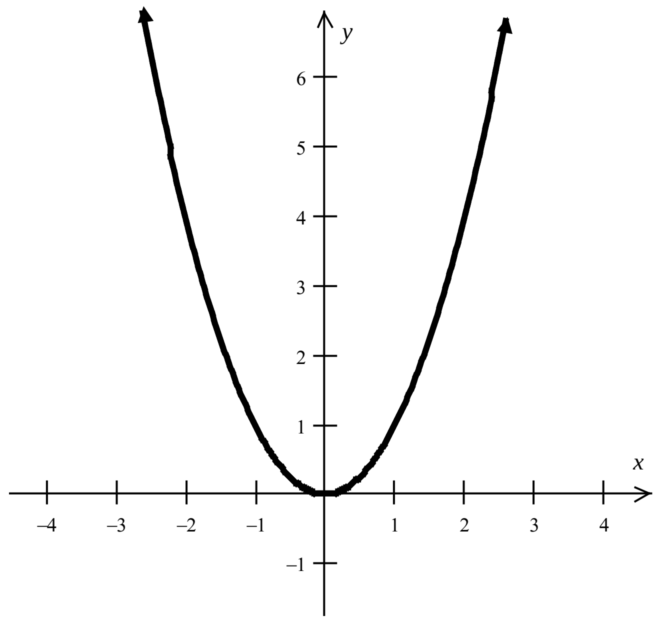
# Section G - Graphing and Transformations

Graphs of Eight Basic Functions

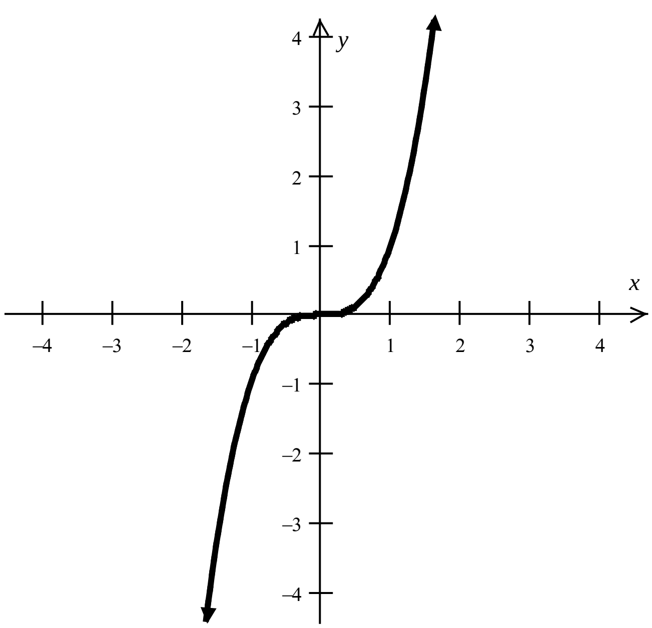
$f(x) = x$



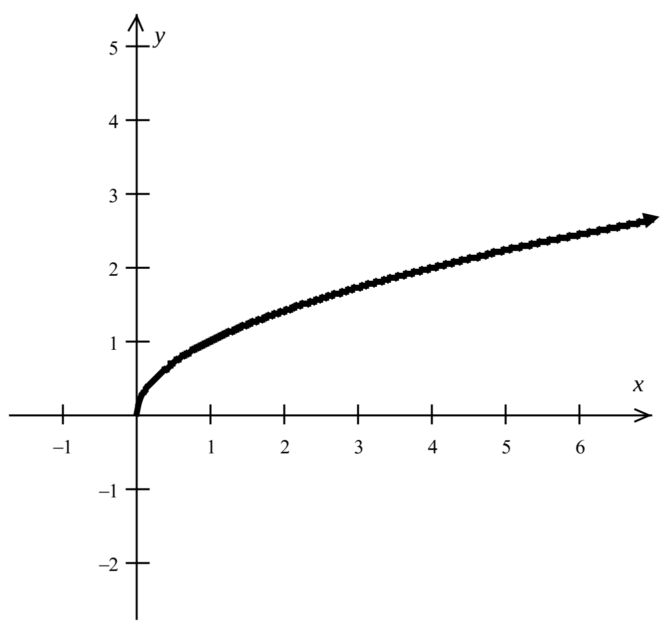

$f(x) = x^2$



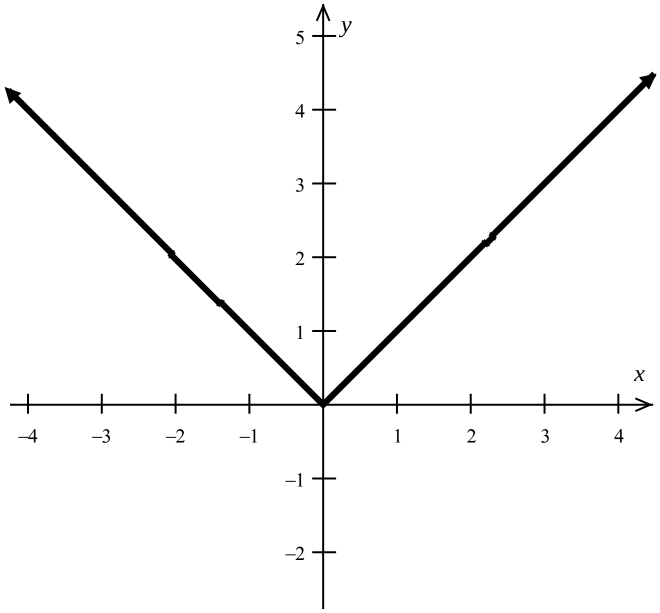

$f(x) = x^3$



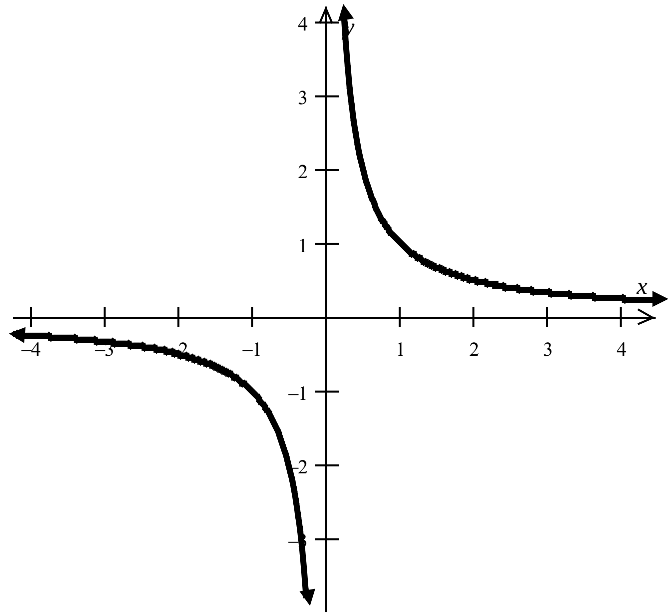

$f(x) = \sqrt{x}$



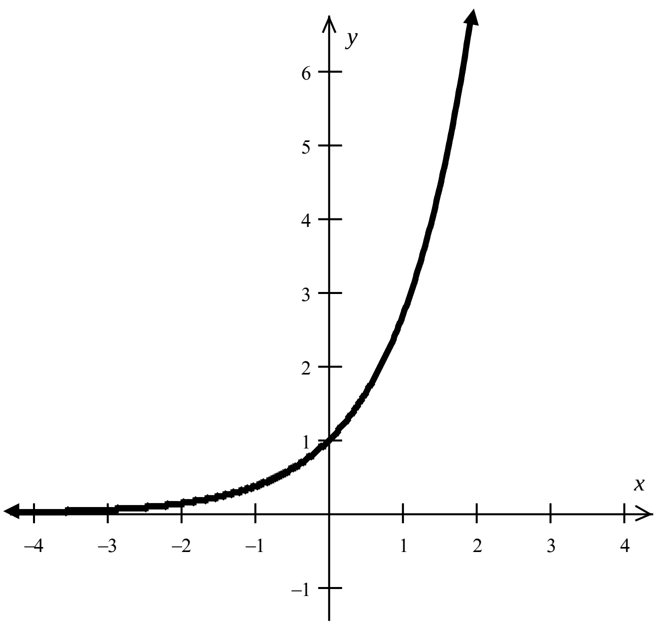
$f(x) = |x|$



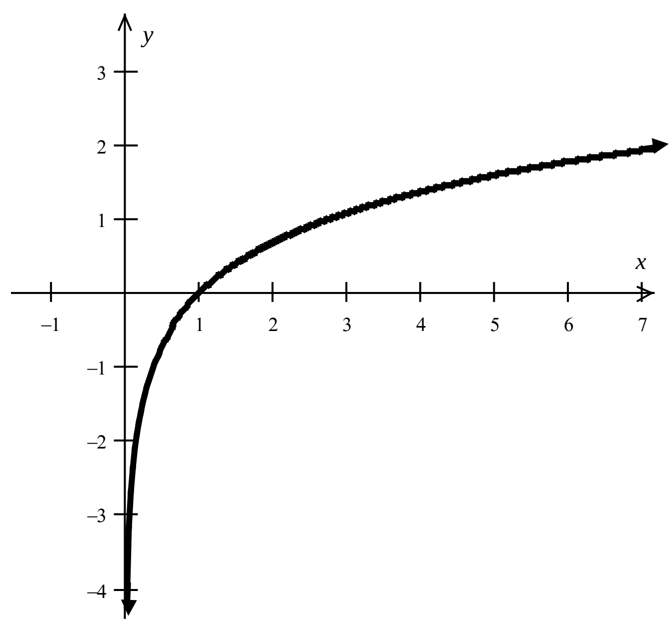

$f(x) = \frac{1}{x}$




$f(x) = e^x$




$f(x) = \ln x$

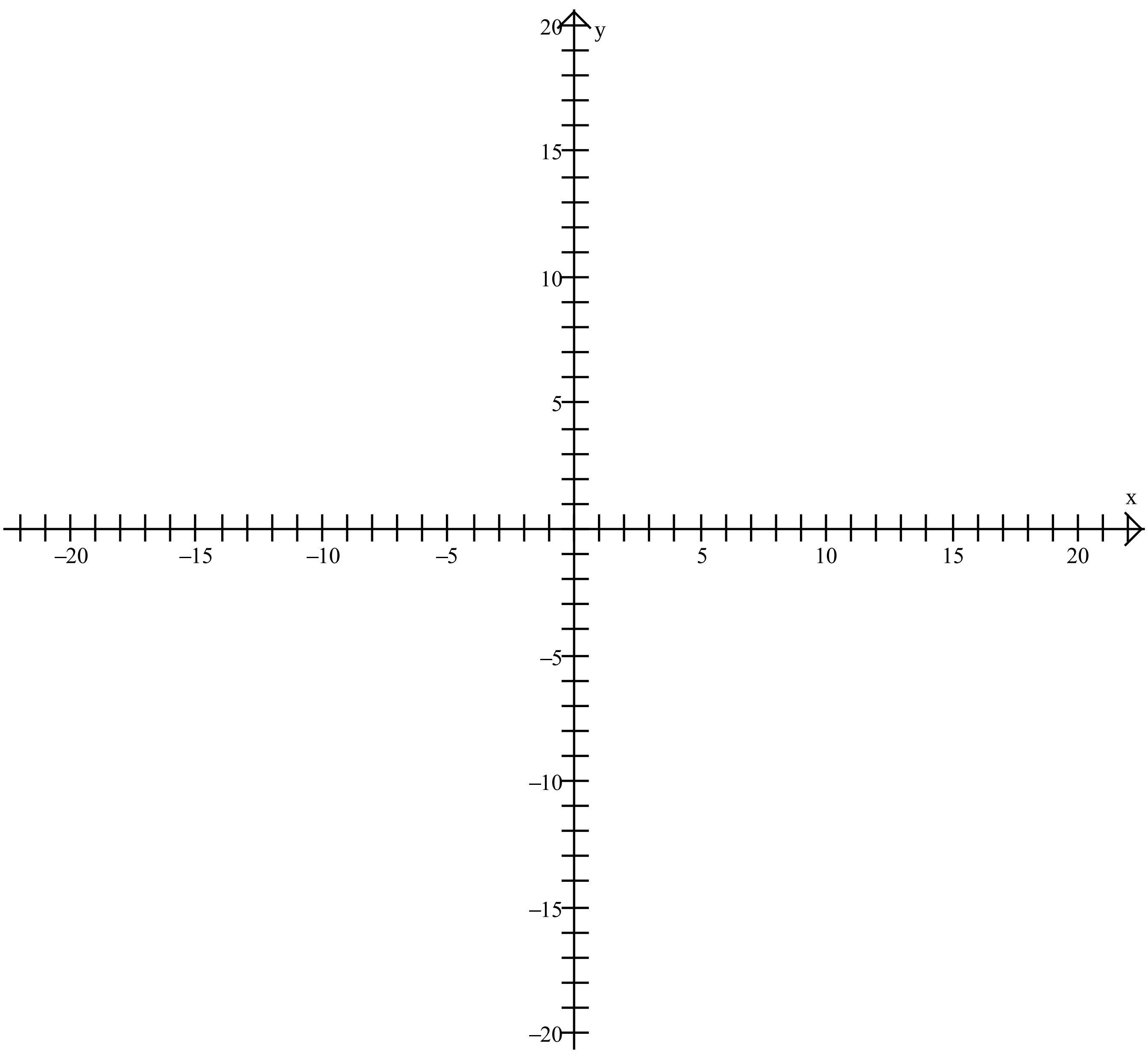



<u>Transformations ($c$ is a positive number)</u>

| | |
|---|---|
| $f(x-c)$ | Horizontal shift $c$ units to the right. |
| $f(x+c)$ | Horizontal shift $c$ units to the left. |
| $f(x)-c$ | Vertical shift $c$ units down. |
| $f(x)+c$ | Vertical shift $c$ units up. |
| $-f(x)$ | Reflection about the x-axis. |
| $f(-x)$ | Reflection about the y-axis. |

Examples: without the use of a calculator, graph the basic function and its transformations on the same set of axes. Find at least three points for each graph. Include the table of points that you are using for your graph. Use different colors for each transformation. In the same color, state the domain and range of each graph.

$f(x)=\sqrt[4]{x}$ $\qquad g(x)=\sqrt[4]{x-3}$ $\qquad h(x)=-\sqrt[4]{x}+6$ $\qquad j(x)=\sqrt[4]{-x}-7$



# Section H - Trigonometry

The Six Trigonometric Functions

$\sin\theta = \dfrac{opposite}{hypotenuse}$ $\cos\theta = \dfrac{adjacent}{hypotenuse}$ $\tan\theta = \dfrac{opposite}{adjacent}$

$\csc\theta = \dfrac{1}{\sin\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\cot\theta = \dfrac{1}{\tan\theta}$

**S**
**O**
**H**
**C**
**A**
**H**
**T**
**O**
**A**

Tangent Identities

$\tan x = \dfrac{\sin x}{\cos x}$ $\cot x = \dfrac{\cos x}{\sin x}$

$\cot^2 x + 1 = \csc^2 x$

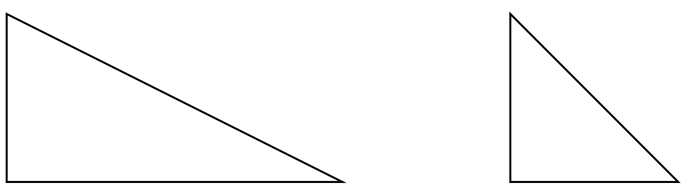
Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$

Symmetry Identities

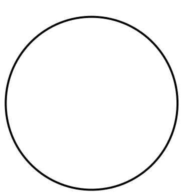
$\sin(-x) = -\sin x$ $\cos(-x) = \cos x$ $\tan(-x) = -\tan x$

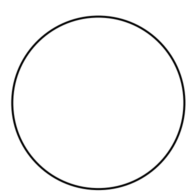
Special Triangles You Need To Know



Evaluating Trigonometric Functions

Coterminal angles are angles that end up in the same place in the $360°$, $2\pi$ radians circle.





When evaluating a trigonometric function at one of these points, use the *x* and *y* coordinates.

When evaluating a trigonometric function anywhere else, use one of the bowtie triangles.

Examples: Find the values of the six trigonometric functions for the given angle, $\theta$

$\theta = \frac{5\pi}{3}$

$\theta = -\frac{3\pi}{2}$

$\theta = \frac{7\pi}{6}$

$\theta = -\frac{5\pi}{4}$

Given $g(x) = \cos^2\left(\frac{\pi}{8}x\right) + \sin\left(\frac{\pi}{4}x\right)$, then $g(6) =$

Given $f(t) = \sin\left(\frac{t}{12}\right) - \tan^3\left(\frac{t}{6}\right)$, then $f(10\pi) =$

Examples: Find all solutions to each equation for $0 \le x \le 2\pi$

$2\sin^2 x + \sin x = 1$

$2\cos^2 x - \cos x = 0$

$\csc^2 x - 4 = 0$

$\sin 2x + \cos 2x = 0$

Examples: Find all solutions to each equation for $0 \le x \le \pi$

$\sec^2 x + 3\sec x = -2$

$2\sin^2 x + \sin x = 0$

$2\cos 3x - \sqrt{2} = 0$

$\tan^2 2x - 3 = 0$

## Section I - Direct and Inverse Proportionality (Variation)

Two variables, $y$ and $x$, are **directly proportional** (also may be worded **proportional** or **varies directly**) if there is a non-zero constant, $k$, such that $y = kx$

Two variables, $y$ and $x$, are **inversely proportional** (also may be worded **varies inversely**) if there is a non-zero constant, $k$, such that $y = \frac{k}{x}$

***k*** is called the **constant of proportionality** (also may be referred to as the **constant of variation**)

**Steps for solving proportionality problems**

**1. Write an equation using *k* as the constant of proportionality that accurately describes the type of variation.**

**2. Using the given information, solve for the constant of proportionality, *k*.**

**3. Substitute the constant of proportionality, *k*, into the original equation from part a) to represent the exact relationship between the given variables.**

**4. Use the equation from part c) to answer the question at the end of the problem.**

Suppose the rate, $R$, that water flows out of a dam is proportional to the annual amount of snowfall, $A$, in the mountains above the dam. Water flows out of the dam at a rate of 10 billion gallons per day when there are 40 inches of snowfall in a year in the mountains above the dam. At what rate will water flow out of the dam when there are 60 inches of snowfall in a year in the mountains above the dam?

The distance, $y$, that an object falls is directly proportional to the square of the time, $t$. A certain object falls 64 feet in 2 seconds. How far will this object fall in 5 seconds?

Boyle's law states that the pressure, *P*, of a gas is inversely proportional to the volume, *V*, of the gas if the temperature remains constant. At a certain temperature, when the gas is in a 4 liter container, the pressure of the gas is 12 pounds per square inch(psi). At the same temperature, what will the pressure of the gas be if transferred to a $\frac{1}{3}$ liter container.

Suppose a tennis player has a return of serve percentage, *P*, that is inversely proportional to the square of the speed of the serve, *S*. When the speed of the serve is 90 mph, the tennis player's return of serve percentage is 70%. What will be the tennis player's return of serve percentage for serves that have a speed of 100 mph?

Suppose the cost, *C*, to manufacture a television is directly proportional to the square root of the size of the screen, *L* (measured by the length of the diagonal). It costs $120 to manufacture a television with a 25 inch screen. How much does it cost to manufacture a television with a 36 inch screen?

The rate, *R*, at which a virus spreads among a population of $X$ people is proportional to the product of the number of people who have contracted the virus and the number of people who have not contracted the virus. *V* denotes the number of people who have contracted the virus. Write an equation using $k$ as a constant that could be used to model this situation.

The surface area, *S*, of a sphere, is related to the radius, *r*, of the sphere, by the formula $S = 4\pi r^2$.

a) Describe in words the relationship between the surface area and the radius of a sphere.
b) What is the constant of proportionality?
c) If the radius of the sphere is tripled, what will happen to the surface area of the sphere?

**AP Calculus AB Summer Review Packet** **Name________________________**

**(200 homework points – Due Monday 8/21/17, 20 points extra credit if <u>completed</u> and turned in by 8/19/17)**

This packet is designed to help you review and build upon some of the important mathematical concepts and skills that you have learned in your previous mathematics classes that you will be using in AP Calculus AB. Before you work on each section, go to my website www.mrdemsey.com and watch the corresponding videos and fill out the prerequisite notes for that section. The first exam of the year will be during the second week of school and will be largely based on the concepts and methods you will be practicing in the problems of this packet.

PUT ALL WORK AND ANSWERS ON SEPARATE SHEETS OF PAPER

## Section A – Linear Equations

For problems 1-8, based on the information given, write the linear equation in:

c) point-slope form

d) slope-intercept form

1. The line with a slope of $3$ passing through the point $(4,-2)$

2. The line that passes through the points $(2,4)$ and $(4,-5)$

3. The line that contains the point $(-3,2)$ and is parallel to $x+y=7$

4. The line perpendicular to $2x-4y=8$ containing the point $(1,-2)$

5. The line with a slope of $m=-\frac{2}{3}$ and an *x*-intercept of 5.

6. The line with a y-intercept of $-2$ and passing through the point $(-1,4)$

7. The line parallel to $3y-5x=3$ and passing through $(-4,-8)$

8. The line that passes through the point $(-2,7)$ and perpendicular to $5x+2y=-7$

9. A horizontal line that goes through the point $(-3,6)$

   a) What is the slope of the line?

   b) Write the equation of the line.

   c) Write the equation of the line perpendicular to the line that passes through the point $(-1,7)$

10. A vertical line that goes through the point $(5,-2)$

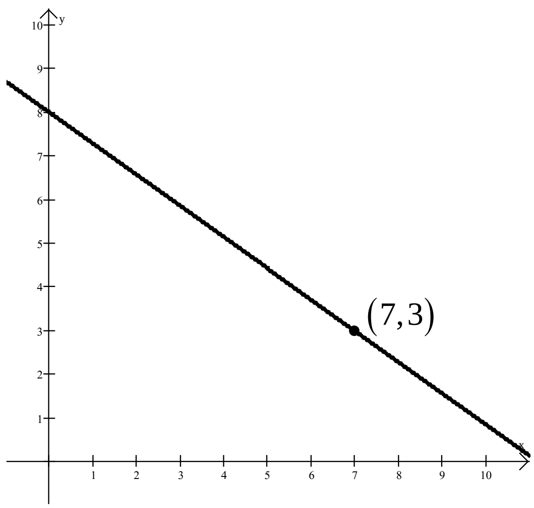
   a) What is the slope of the line?

   b) Write the equation of the line.

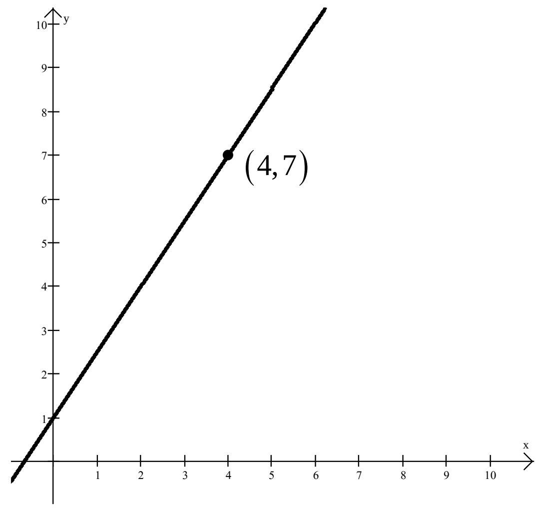
   c) Write the equation of the line parallel to the line that passes through the point $(-4,2)$

For problems 11-14, write the linear equation in slope-intercept form using the given graph.

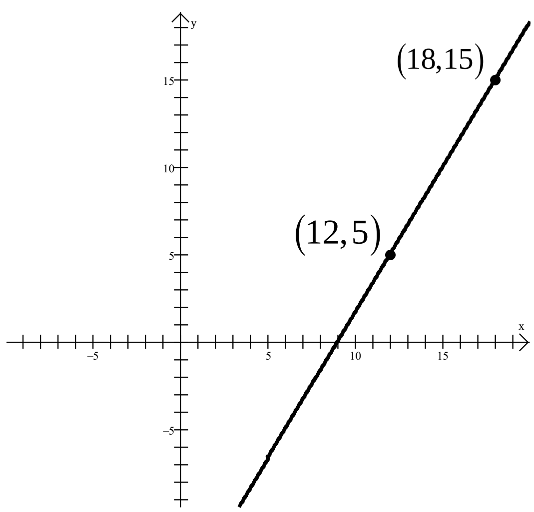
11.



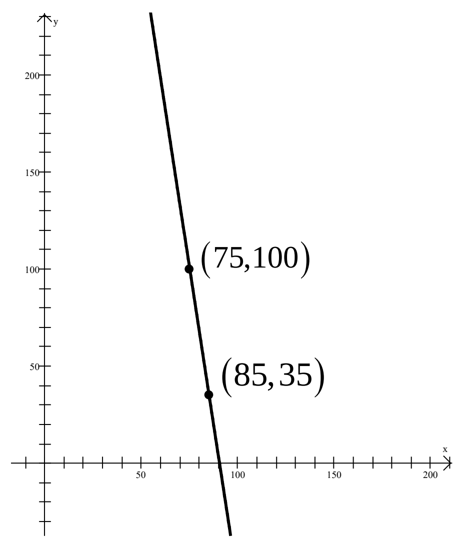

12.




13.




14.




15. The functions *f*, *g*, and *h* are continuous functions that have the values given by the tables below. Determine whether each function could be a linear equation and explain why or why not.

a)

| $x$ | $f(x)$ |
|---|---|
| 3 | 2 |
| 5 | 7 |
| 9 | 17 |

b)

| $x$ | $g(x)$ |
|---|---|
| 2 | 11 |
| 6 | 13 |
| 12 | 17 |

c)

| $x$ | $h(x)$ |
|---|---|
| 1 | 23 |
| 5 | 17 |
| 13 | 5 |

## Section B Factoring , Simplifying, and Solving Equations and Inequalities Using Factoring

For problems 1-4, factor completely.

1. $16x^2 - 9$  2. $9x^2 - 3x - 2$  3. $6x^3 - 17x^2 + 5x$  4. $2t^5 - 32t$

For problems 5-8, simplify each expression using factoring.

5. $\dfrac{x^2 - 4x - 5}{x^2 + 2x + 1}$  6. $\dfrac{x^3 - 4x^2 - 32x}{64 - x^2}$  7. $\dfrac{x^2 - 2x - 8}{x^3 + x^2 - 2x}$  8. $\dfrac{3x(x-1) - 2x^2}{2x^2 - 5x - 3}$

For problems 9-12, solve the following equations by factoring.

9. $x^3 - 16x = 0$  10. $t^2 + 3t - 4 = 14$  11. $(x+1)^2(x-2) + (x+1)(x-2)^2 = 0$  12. $2x^2 - 12 = -5x$

For problems 13-16, solve the following inequalities by factoring. Write your answer using inequality and interval notation.

13. $x^2 + 7x + 12 > 0$  14. $x^3 - 8x^2 < 9x$  15. $\dfrac{9 - x^2}{x} \geq 0$  16. $\dfrac{x^2 + 13x - 30}{x^2 - 25} < 0$

## Section C Exponents, Radicals, and Simplifying

For problems 1-5, write without fractional or negative exponents.

1. $f(x) = 2x^{-3}$  2. $3x^{\frac{1}{3}}y^{-\frac{1}{2}}$  3. $y = 27^{1/3}x^{3/4}$  4. $y = \dfrac{(x-3)^{-2}}{(2x+1)^{-3}}$  5. $y = 25^{\frac{3}{2}}2^{-3}x^{-2}$

For problems 6-9, write without fractional or negative exponents in order to find the following:

6. $f(x) = x^{\frac{5}{2}} - x^{-1}$  7. $g(x) = (x+19)^{\frac{2}{3}} - x^{\frac{5}{3}}$  8. $h(x) = 3x^{-\frac{1}{4}} - 16x^{-\frac{3}{2}}$  9. $v(t) = \dfrac{3t^{\frac{1}{3}}}{2} + \dfrac{5t^{\frac{1}{2}}}{2}$

$f(4) =$  $g(8) =$  $h(16) =$  $v(64) =$

For problems 10-14, factor and then simplify. Your final answer should not contain any negative or fractional exponents.

10. $f(x) = 4x^{-3} + 2x - 18x^{-2}$  11. $f(x) = 5x^2(x-2)^{-1/2} + (x-2)^{1/2}3x$

12. $f(x) = 6x(2x-1)^{-1} - 4(2x-1)$  13. $y = e^{-x} - xe^{-x} + 2x^2e^{-x}$

14. $g(x) = 10x^2 \cdot \dfrac{1}{5}(3x^2 - 4)^{-\frac{4}{5}} \cdot 6x + (3x^2 - 4)^{\frac{1}{5}} \cdot 20x$

For problems 15-19, you are to demonstrate that the two sides of the equation are equivalent by simplifying only the **left side** to make it look identical to the right side.

15. $x \cdot \dfrac{1}{2}(x^2+5)^{-\frac{1}{2}}(2x) + (x^2+5)^{\frac{1}{2}} = \dfrac{2x^2+5}{\sqrt{x^2+5}}$

16. $x^2 \cdot 4(x-2)^3 + (x-2)^4(2x) = 2x(x-2)^3(3x-2)$

17. $x \cdot 3(3x-9)^2(3) + (3x-9)^3 = 27(x-3)^2(4x-3)$

18. $x \cdot \dfrac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x) + (1-x^2)^{\frac{1}{2}} = \dfrac{1-2x^2}{\sqrt{1-x^2}}$

19. $x^2 \cdot \dfrac{1}{2}(9-x^2)^{-\frac{1}{2}}(-2x) + (9-x^2)^{\frac{1}{2}}(2x) = \dfrac{3x(6-x^2)}{\sqrt{9-x^2}}$

## Section D Manipulating Equations and Solving Systems of Equations

In problems 1-9, solve for $y$ in terms of the other variable.

1. $\frac{1}{2-y} = x^3 + 4x$
2. $xy - 6x^2 = 7y - 5x + 4$
3. $3\ln y = 6x^2 + 3x - 12$
4. $e^y - 6 = x$
5. $\frac{2y+5t}{y} = \frac{3t}{2}$
6. $\frac{1}{y^2} = x^2 + 5x - 7$
7. $2\ln(y+3) = 4x^2 - 7$
8. $-\frac{1}{2y^2} = \frac{x^2}{2} - 5$
9. $\sin(2y-5) = x^4 - 7x$
10. $e^{\sqrt{2y-5}} - 3 = 4x$
11. $\cos^2 y = e^x - 7$
12. $\frac{1}{3}\ln(3y+7) = \frac{x^3}{3} - 5x + 2$

For problems 13-20, solve the system of equations using substitution or elimination.

13. $\begin{aligned} 2x+3y&=-2 \\ 5x+2y&=-27 \end{aligned}$
14. $\begin{aligned} 5a+3b&=9 \\ 2a-4b&=14 \end{aligned}$
15. $\begin{aligned} y&=3x-10 \\ x&=12-4y \end{aligned}$
16. $\begin{aligned} 2b+5&=a \\ 2b+3a&=13 \end{aligned}$
17. $\begin{aligned} y&=x^2-4x-9 \\ y&=6x+15 \end{aligned}$
18. $\begin{aligned} y&=3x^2-5x-4 \\ y&=x^2+8x+11 \end{aligned}$
19. $\begin{aligned} y&=3\sqrt{x} \\ y&=x+2 \end{aligned}$
20. $\begin{aligned} y&=\sqrt{4-x^2} \\ y&=2-x \end{aligned}$

## Section E Symmetry and Intercepts

Check for symmetry with respect to each axis and the origin.

1. $xy - \sqrt{4-x^2} = 0$
2. $xy^2 = -10$
3. $xy = 1$
4. $y = x^4 + x^2$
5. $\frac{x}{x^2+1} = y$

Find the $x$ and $y$ intercept(s) , if they exist, for each of the following.

6. $2y = 6x + 4$
7. $x = y^2 - 4$
8. $y = \frac{x-1}{x-2}$
9. $x^2y - x^2 + 4y = 0$
10. $y = \frac{10}{x^2+1}$

## Section F Functions

1. Given the function: $f(x) = -2x(x+3)^4(x-2)^3$

   a) What are the zeros of the function? b) $f(-1) =$ c) $f(x+1) =$ d) $f(e^x) =$

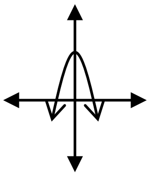
2. Given $f(x) = 2x+1$ and $g(x) = 1 - x^2$ , find each of the following:

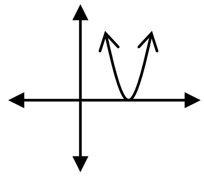
a) $f(g(0))$ b) $g(g(2))$ c) $g(f(x))$ d) $f(g(x))$ e) $f(f(x))$

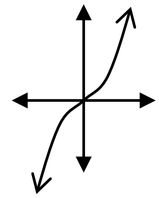
3. If $f(x) = 5x+3$ and $g(x) = 2x^2 - 3x + 4$, find the following:

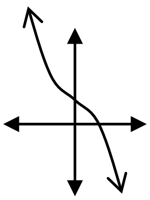
a) $g(f(-1))$ b) $g(\ln x)$ c) $f(\cos x)$ d) $g(\sin x)$ e) $f(g(\ln x))$

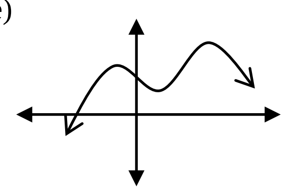
4. For each of the following functions, state if they are even, odd, or neither.

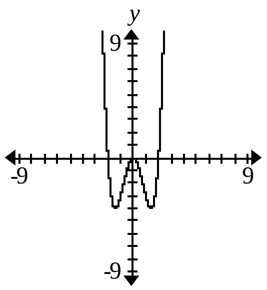
a) 

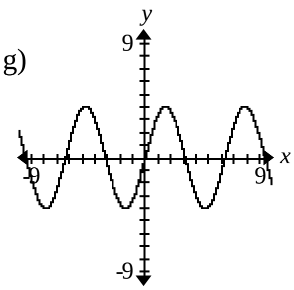
b) 

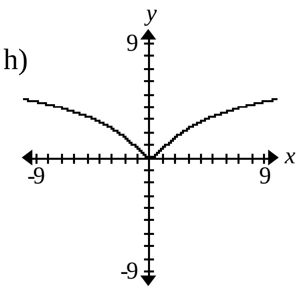
c) 

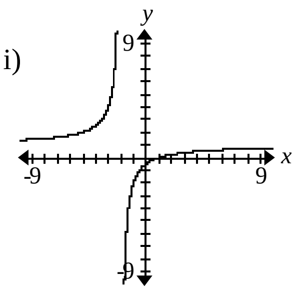
d) 

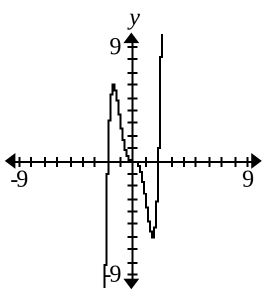
e) 

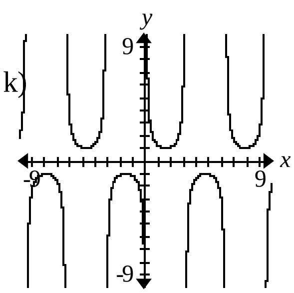
f) 

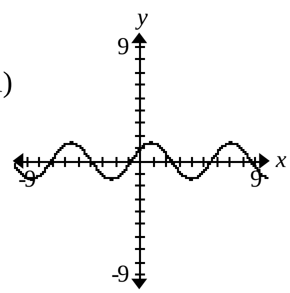

g) 

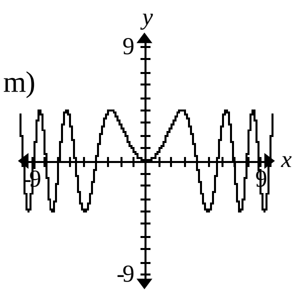

h) 


i) 


j) 


k) 


l) 


m) 


For problems 5-8, determine algebraically whether the following functions are even, odd, or neither.

5. $f(x) = -x^5 + 3x^3 - 2x$ 6. $g(x) = x - x^4$ 7. $h(x) = \dfrac{1}{x^2 - 4}$ 8. $j(x) = \dfrac{x^3 - x}{2x^2 + 1}$

9. Let $g(0) = 1$, $g(1) = -3$, $g(2) = 5$, $g(7) = 2$, $h(1) = 7$, $h(2) = 1$, $h(5) = 0$. Evaluate the following:

a) $(g \circ h)(2)$ b) $(h \circ g)(2)$ c) $g(g(h(1)))$

d) $h(g(h(5)))$ e) $g^{-1}(5)$ f) $h^{-1}(0)$

For problems 10-17, find the domain of each function. Give your answer using interval notation.

10. $h(x) = \sqrt{4 - 2x}$ 11. $g(x) = \ln(x + 3)$ 12. $f(x) = \dfrac{2}{x - 1}$ 13. $h(x) = x^2 + 4$

14. $f(x) = \sqrt{16 - x^2}$ 15. $g(x) = \dfrac{2x - 3}{\sqrt{x^2 + 2x - 24}}$ 16. $f(x) = \ln(x^2 - 9)$ 17. $h(x) = \ln\left(\dfrac{x - 4}{x + 3}\right)$

For problems 18-26, use the table below to find the following:

| $x$ | -6 | -4 | -2 | 0 | 2 | 4 | 6 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 7 | 2 | 1 | -1 | 3 | 5 | 10 |
| $g(x)$ | -4 | -2 | 0 | 2 | 4 | 6 | 8 |
| $h(x)$ | 2 | 0 | -1 | -4 | -2 | 1 | 4 |

18. $f(-6)$

19. $f(g(0))$

20. $f(h(-6))$

21. $g(h(f(-4)))$

22. $f^{-1}(10)$

23. $h^{-1}(g(-4))$

24. $h^{-1}(g^{-1}(2))$

25. $g^{-1}(h(f^{-1}(7)))$

26. Which function $f$, $g$, or $h$ could be a linear function? Explain why.

27. Find the domain of $g(x) = \dfrac{\sqrt{x+2}}{x^2 - x}$

28. What is the domain of the function: $\dfrac{3x+1}{\sqrt{x^2+x-2}}$

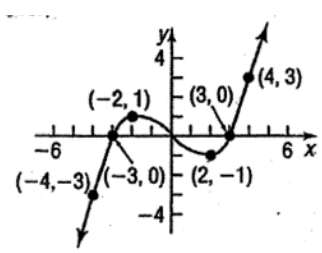
29. The function $f$ is given by $f(x) = e^{(x-11)} - 8$. Find $f^{-1}(x)$ and its domain.

30. Let $f(x) = \dfrac{3x+7}{x-2}$. Find $f^{-1}(x)$, the inverse of $f(x)$

31. Given $f(x) = 2x^3 + 1$, find $f^{-1}(x)$, the inverse of $f(x)$.

32. If $f(x) = \dfrac{4x}{x+1}$, find $f^{-1}(x) =$

33. Use the graph below of the function $f$ to answer the questions below




a) Find the zeros of the function.

b) Is the function even, odd, or neither?

c) $f(-2) =$

d) $f^{-1}(3) =$

# Section G – Graphing and Transformations

For problems 1 -7, without the use of a calculator, graph the basic function and its transformations on the same set of axes. Find at least three points for each graph. Include the table of points that you are using for your graph. Use different colors for each transformation. In the corresponding color, state the domain and range of each graph.

1. $f(x)=x^2$ $\quad g(x)=x^2-3$ $\quad h(x)=(x-3)^2$ $\quad j(x)=-3x^2$

2. $f(x)=|x|$ $\quad g(x)=|x+4|$ $\quad h(x)=2|x|+4$ $\quad j(x)=|x-2|-1$

3. $f(x)=e^x$ $\quad g(x)=e^x+2$ $\quad h(x)=e^{-x}$ $\quad j(x)=-e^{x-2}$

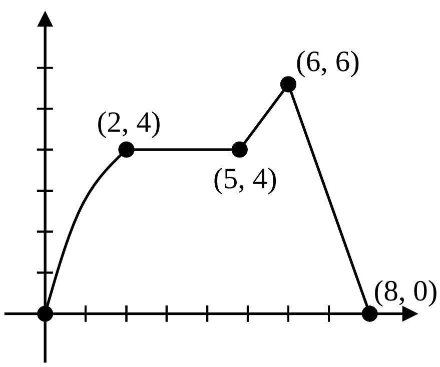
4. $f(x)=x^3$ $\quad g(x)=x^3+2$ $\quad h(x)=(x-4)^3$ $\quad j(x)=-(x+2)^3+1$

5. $f(x)=\ln x$ $\quad g(x)=\ln(x+3)$ $\quad h(x)=\ln x+3$ $\quad j(x)=-\ln(x-2)$

6. $f(x)=\frac{1}{x}$ $\quad g(x)=\frac{1}{x-2}$ $\quad h(x)=-\frac{1}{x}$ $\quad j(x)=\frac{1}{x+4}-2$

7. $f(x)=\sqrt{x}$ $\quad g(x)=\sqrt{-x}+3$ $\quad h(x)=-\sqrt{x+1}$ $\quad j(x)=\sqrt{x-3}+4$

For problems 8-11,use the graph of $f$ below to graph the transformations.
Then state the domain and range of each.




8. $f(x)+2$
9. $-f(x)$
10. $f(x-1)$
11. $f(x+1)-2$

## Section H - Trigonometry

For problems 1-18, without a calculator, determine the exact value of each expression.

1. $\cos 0$ 2. $\sin \frac{\pi}{2}$ 3. $\csc \frac{3\pi}{4}$ 4. $\cos \pi$

5. $\cos \frac{7\pi}{6}$ 6. $\cos\left(-\frac{\pi}{3}\right)$ 7. $\tan \frac{7\pi}{4}$ 8. $\tan \frac{\pi}{6}$

9. $\sin \pi$ 10. $\sin\left(\frac{3\pi}{2}\right)$ 11. $\cot\left(\frac{5\pi}{4}\right)$ 12. $\cos\left(-\frac{2\pi}{3}\right)$

13. $\cos\left(\frac{7\pi}{4}\right)$ 14. $\sin\left(\frac{2\pi}{3}\right)$ 15. $\tan\left(\frac{5\pi}{6}\right)$ 16. $\sin\left(\frac{11\pi}{6}\right)$

17. $\sin^2\left(\frac{\pi}{4}\right) - \cos^2\left(\frac{\pi}{6}\right)$ 18. $3\sin\left(\frac{\pi}{2}\right) - \cos\left(\frac{\pi}{3}\right)$

19. Given $f(x) = \cos(\pi x) + \sin\left(\frac{\pi}{12}x\right)$, then $f(15) =$ 20. Given $g(x) = \tan\left(\frac{\pi}{3}x\right) + \cos\left(\frac{\pi x}{6}\right)$, then $g(4) =$

21. Given $h(x) = \sin^3(3x) + \cos^2\left(\frac{x}{2}\right)$, then $h\left(\frac{\pi}{2}\right) =$ 22. Given $f(t) = \sin^2\left(\frac{t}{4}\right) - \tan\left(\frac{t}{12}\right)$, then $f(3\pi) =$

For problems 23-30, without a calculator, find all solutions to each equation for $0 \le x \le 2\pi$.

23. $4\cos^2 x + 4\cos x = -1$ 24. $2\sin^2 x + 3\sin x + 1 = 0$

25. $2\sin^2 x - \sin x = 0$ 26. $2\sin 2x - \sqrt{3} = 0$

27. $\sec^2 x - 2\sec x = 0$ 28. $\tan^2 x + \tan x = 0$

29. $2\cos^2 x + \cos x - 1 = 0$ 30. $\sin x - \cos x = 0$

For problems 31-38, without a calculator, find all solutions to each equation for $0 \le x \le \pi$.

31. $2\cos^2 3x - \sqrt{3}\cos 3x = 0$ 32. $\tan^2 x - 1 = 0$

33. $2\cos 2x + \sqrt{3} = 0$ 34. $2\sin 2x - \sqrt{2} = 0$

35. $\sin 3x + \cos 3x = 0$ 36. $2\cos^2 x + \cos x = 0$

37. $4\cos^2 x - 3 = 0$ 38. $3\csc^2 x - 4 = 0$

39. Identify which of the six trig functions are

a) even b) odd c) neither

(Hint: draw a quick sketch of their basic graphs)

# <u>Section I – Direct and Inverse Proportionality</u>

For problems 1-10, find the following:
a) Write an equation using *k* as the constant of proportionality that accurately describes the type of variation.
b) Using the given information, solve for the constant of proportionality, *k*.
c) Substitute the constant of proportionality, *k*, into the original equation from part a) to represent the exact relationship between the given variables.
d) Use the equation from part c) to answer the question at the end of the problem.

1. When swimming underwater, the pressure, *P*, in a person's ears is directly proportional to the depth, *h*, at which the person is swimming. At 10 feet deep, the pressure is 4.3 pounds per square inch (psi). What is the pressure at a depth of 20 feet?

2. Suppose the velocity, *v*, of a particle traveling in a straight line, is proportional to the square root of the time, *t*. The velocity is 78 mm/sec at time $t = 9$ seconds. Determine the velocity of the particle at $t = 49$ seconds.

3. Suppose the number of people, *P*, at an outdoor concert is directly proportional to the cube root of the number of albums, *A*, sold by the performing artist. There are 20,000 people at a concert for an artist who has sold 262,144 albums. How many people would attend a concert for an artist who has sold 1,000,000 albums?

4. Suppose a basketball player has a shooting percentage, *P*, that is inversely proportional to the square root of her distance from the basket, *d*. When her distance from the basket is 9 feet, her shooting percentage is 72%. What will be her shooting percentage when she is 16 feet from the basket?

5. The illumination, *I*, from a light source is inversely proportional to the square of the distance, *x*, from the light source. When a book is 2 meters from a certain lamp, the illumination is 270 lux (lux is a unit for illumination meaning lumens per square meter). What will be the illumination if the book is 3 meters from this lamp?

6. Boyle's law states that the pressure, *P*, of a gas is inversely proportional to the volume, *V*, of the gas if the temperature remains constant. At a certain temperature, when the gas is in a 2 liter container, the pressure of the gas is 10 pounds per square inch(psi). At the same temperature, what will the pressure of the gas be if transferred to a ½ liter container.

7. According to Hook's Law, the force, *F*, required to stretch a spring is directly proportional to the distance, *x*, the spring is stretched. For a certain spring, a 10 lb force stretches a spring 1 inch. What force will be required to stretch this spring 3 inches?

8. The period, *P*, (the time required for one complete oscillation) of a simple pendulum is directly proportional to the square root of the length, *L*, of the pendulum. The length of a certain pendulum is 9 feet and has a period of 6 seconds. How long will the pendulum need to be in order for its period to be 10 seconds?

9. The kinetic energy, *E*, of a moving object is directly proportional to the square of its velocity, *v*. The velocity of a certain object is 4 meters per second and its kinetic energy is 24 joules. What velocity will be necessary in order to for this object's kinetic energy to equal 44 joules?

10. Suppose the cost, *C*, to manufacture a smartphone is proportional to the product of the gigabytes of internal storage, *G*, and the area of the screen, *A*. It costs $36 to manufacture a smartphone that has 16 gigabytes of internal storage and a screen size of 24 $cm^2$. How much will it cost to manufacture a smartphone that has 32 gigabytes of internal storage and a screen size of 28 $cm^2$?

11. The rate, $R$, at which a rumor spreads among a population of $P$ people is proportional to the product of the number of people who have heard the rumor and the number of people who have not heard the rumor. $Y$ denotes the number of people who have heard the rumor. Write an equation with $k$ as the constant of proportionality that could be used to model this situation.

12. The volume, $V$, of a sphere, is related to the radius, $r$, of the sphere, by the formula $V = \frac{4}{3}\pi r^3$.

d) Describe in words the relationship between the volume and the radius of a sphere.
e) What is the constant of proportionality?
f) If the radius of the sphere is doubled, what will happen to the volume of the sphere?

13. The area, $A$, of a circle, is related to the radius, $r$, of the circle, by the formula $A = \pi r^2$.

a) Describe in words the relationship between the area and the radius of a circle.
b) What is the constant of proportionality?
c) If the radius of the circle is quadrupled, what will happen to the area of the circle?

14. The circumference, $C$, of a circle, is related to the radius, $r$, of the circle, by the formula $C = 2\pi r$.

a) Describe in words the relationship between the circumference and the radius of a circle.
b) What is the constant of proportionality?
c) If the radius of the circle is tripled, what will happen to the circumference of the circle?

15. The law of gravitational attraction states the gravitational force, $F$, between two bodies is directly proportional to the product of the masses, $m_1$ and $m_2$, of the two bodies, and inversely proportional to square of the distance, $r$, between the two bodies so that the equation that relates these variables is $F = G\frac{m_1 m_2}{r^2}$.

The constant of proportionality, $G$ (instead of $k$), is called the gravitational constant.

a) If the distance between two bodies is tripled, what will happen to the gravitational force?

b) If the mass of one body is quadrupled and the distance between the objects is doubled, what will happen to the gravitational force?